# Chapter 20
# Research of Precise Timing in Single Receiver Pseudorange Positioning Based on GPS System

**Hehuan Zhu, Xiangyang Wu, Weirong Chen and Shuguo Pan**

**Abstract** Timing service is one of the significant basic services in GPS system. Principles and mathematical models of the precise timing in single station pseudorange positioning were provided in this paper. And the automatic batch of timing procedures was developed through the C++ language. The experimental results show that, the precision of timing in single station pseudorange positioning based on GPS system can reach nanosecond level. In addition, the precision can be higher when using carrier smoothing pseudorange method. The precise timing services in GPS system has broad prospect in communications, electric power and other industries.

**Keywords** Precise timing · GPS system · Single station pseudorange positioning · Precision

## 20.1 Introduction

High-precision timing is significant to the operation of military, communication, power and transportation system, etc. It has universal application in national defense building and economic construction. With the rapid development of modern society, the requirements of timing precision are increasing. The research of precise timing based on GPS system is increasingly important, because GPS system has developed relatively more perfectly, and its precision is relatively higher.

H. Zhu (✉) · X. Wu · W. Chen
School of Transportation, Southeast University, No. 2 Sipailou, Nanjing 210096, China
e-mail: zhuhehuan328@163.com

S. Pan
School of Instrument Science and Engineering, Southeast University, No. 2 Sipailou, Nanjing 210096, China

J. Sun et al. (eds.), *China Satellite Navigation Conference (CSNC) 2013 Proceedings*, Lecture Notes in Electrical Engineering 243,
DOI: 10.1007/978-3-642-37398-5_20, 

The precision of pseudorange observations is not high, while the determination of ambiguity is a major problem when using carrier phase observations. However, using carrier phase smoothed pseudorange method can not only improve the accuracy of positioning and timing, but also avoid the determination of ambiguity.

The precision of timing in single station pseudorange positioning based on GPS system can reach nanosecond level. By demonstrated, the precision can be higher when using carrier smoothing pseudorange method.

## 20.2 The Principles of Precise Timing by Pseudorange Positioning Based on GPS System

### 20.2.1 Observation Equations in Pseudorange Positioning

After calculating the satellite coordinates with broadcast ephemeris data, Eq. (20.1) [1] is used as fundamental model of pseudorange positioning.

$$\rho' = \sqrt{[X^j(t^j) - X_k(t_k)]^2 + [Y^j(t^j) - Y_k(t_k)]^2 + [Z^j(t^j) - Z_k(t_k)]^2} + c\delta t_k - c\delta t^j \tag{20.1}$$

According to approximate coordinates and Eq. (20.1), mark that

$$l_p^i = -\frac{X^i - X_p^0}{\rho_{p,0}^i}, m_p^i = -\frac{Y^i - Y_p^0}{\rho_{p,0}^i}, n_p^i = -\frac{Z^i - Z_p^0}{\rho_{p,0}^i} \tag{20.2}$$

The result of linearization can be written as Eq. (20.3) [1],

$$l_p^i \delta X_p + m_p^i \delta Y_p + n_p^i \delta Z_p - c\delta t_p - R_{p,0}^i + (\rho_p^i + c\delta t^i - \delta\rho_{trop}^i - \delta\rho_{ion}^i - \delta\rho_{others}^i) = 0 \tag{20.3}$$

where $\delta\rho_{ion}^i$ is ionospheric delay error (m); $\delta\rho_{trop}^i$ is tropospheric delay error (m); $\delta\rho_{others}^i$ is other errors, such as some effects of earth rotation, tidal, relativistic effects, etc.

Matrix form can be written as Eq. (20.4) [1],

$$\underbrace{\begin{bmatrix} v_p^1 \\ v_p^2 \\ v_p^3 \\ v_p^4 \end{bmatrix}}_{v} = \underbrace{\begin{bmatrix} l_p^1 & m_p^1 & n_p^1 \\ l_p^2 & m_p^2 & n_p^2 \\ l_p^3 & m_p^3 & n_p^3 \\ l_p^4 & m_p^4 & n_p^4 \end{bmatrix}}_{a} \underbrace{\begin{bmatrix} \delta X_p \\ \delta Y_p \\ \delta Z_p \end{bmatrix}}_{\delta x} + \underbrace{\begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}}_{b} \underbrace{\delta\rho_p}_{C\delta tk} - \underbrace{\begin{bmatrix} L_p^1 \\ L_p^2 \\ \cdots \\ L_p^m \end{bmatrix}}_{l} \tag{20.4}$$

If the number of simultaneous observation satellite is equal to 4, it has a unique solution, that is to say $\delta_x = A^{-1}L$. When the number is more than 4, we can use the least square method to calculate, that is to say $\delta_x = (A^TA)^{-1}A^TL$.

### 20.2.2 Carrier Phase Smoothed Pseudorange

Carrier phase smoothed pseudorange method is optimizing pseudorange value on the basis of pseudorange positioning. In this paper, we optimize P1 and P2 [1, 2] simultaneously. Equations are as follows:

$$
\begin{aligned}
\overline{p^j(t_i)} &= \frac{1}{i}p^j(t_i) + \frac{i-1}{i}[(\overline{p^j(t_{i-1})} + \delta p^j(t_{i-1}, t_i)] \\
\overline{p^j(t_i)} &= \frac{1}{i}p^j(t_i) + \frac{i-1}{i}[(\overline{p^j(t_{i-1})} + \lambda(\varphi^j(t_i) - \varphi^j(t_{i-1})]
\end{aligned}
\tag{20.5}
$$

where

$\overline{p^j(t_i)}$ is smoothed pseudorange value on the $i$th epoch;
$p^j(t_i)$ is pseudorange value on the $i$th epoch;
$\overline{p^j(t_{i-1})}$ is smoothed pseudorange value on the ($i-1$)th epoch;
$\lambda\varphi^j(t_i)$ is carrier phase value on the $i$th epoch;
$\lambda\varphi^j(t_{i-1})$ is carrier phase value on the ($i-1$)th epoch.

The timing accuracy is greatly improved when using carrier phase smoothed pseudorange method.

### 20.2.3 Error Models

In this paper, we use broadcast ephemeris data to calculate coordinates and velocity of satellites, then compare calculated clock error, which by pseudorange positioning, with precise ephemeris clock error. Errors take ionospheric delay, tropospheric delay, error by earth rotation and relativistic effects into account.

We use ionosphere-free combination to correct ionospheric delay error. It is demonstrated in Eq. (20.6) [3]:

$$
p = \frac{f_1^2 p_1 - f_2^2 p_2}{f_1^2 - f_2^2} \tag{20.6}
$$

where

$p_1$ means pseudorange value of L1 signal;
$p_2$ means pseudorange value of L2 signal.

Then use Hopfield model to correct tropospheric delay error. It can be written as Eq. (20.7) [3, 4]:

$$\begin{aligned}
\Delta^{trop}(E) &= \Delta d^{trop}(E) + \Delta w^{trop}(E) \\
\Delta w^{trop}(E) &= \frac{10^{-6}}{5}\frac{(-12.96T + 3.718 \times 10^5)}{\sin\sqrt{E^2 + 6.25}}\frac{e}{T^2}11000 \\
\Delta d^{trop}(E) &= \frac{10^{-6}}{5}\frac{77.64}{\sin\sqrt{E^2 + 6.25}}\frac{p}{T}[40{,}136 + 148.72(T - 273.16)]
\end{aligned} \tag{20.7}$$

where

$\Delta^{trop}(E)$ is tropospheric delay error (m);
$\Delta d^{trop}(E)$ is dry delay error (m);
$\Delta w^{trop}(E)$ is wet delay error (m).

Residual error of wet delay error is only several centimeters.

Equation of earth rotation is shown as Eq. (20.8) [4]:

$$\begin{pmatrix} X^{S'} \\ Y^{S'} \\ Z^{s'} \end{pmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} X^s \\ Y^s \\ Z^s \end{pmatrix} \tag{20.8}$$

where $(X^s, Y^s, Z^s)$ is satellite coordinates; $\left(X^{s'}, Y^{s'}, Z^{s'}\right)$ is corrected horizontal coordinates. $\alpha = \omega \cdot \tau$, $\alpha$ is angles of earth when rotate. $\omega$ is earth rotation velocity, $\tau$ is time of satellite signal propagation.

According to satellite coordinates vector and velocity vector, error equation of relativistic effects is established. As shown in Eq. (20.9) [3, 4]:

$$\Delta rela = -\frac{2}{C}X^s \cdot \dot{X}^s \tag{20.9}$$

where $X^s$ is coordinate vector of satellite; $\dot{X}^s$ is velocity vector of satellite.

## 20.3 Experiments and Analysis

In order to analyze and verify method of single receiver pseudorange positioning, this paper adopt the daily observation data of USNO station, which is obtained from IGS website. The receiver of USNO station has atomic clock, which is relatively stable [5, 6]. Timing standard is provided by clock error data of precise ephemeris.

Figure 20.1 shows precise clock error value of USNO by IGS, the interval is 5 min. There are 288 epochs, the average value is $6.241687 \times 10^{-7}$ s.

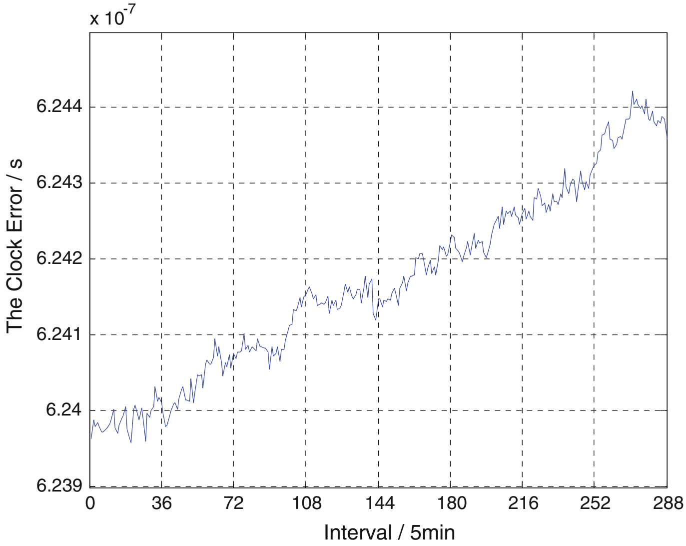


**Fig. 20.1** The clock bias of USNO by IGS

## *20.3.1 Experimental Scheme*

Firstly, we use pseudorange positioning to compute clock error (use the least square method), the average clock error of these 2,880 epochs is $6.268539 \times 10^{-7}$ s, which differs 2.68518 ns with the average value of precision clock error.

Then, compare the clock error with precise clock error by IGS. In addition, interval of pseudorange positioning is 30 s, interval of precise ephemeris file is 5 min, so interval of clock contrast is 5 min. The result is shown in Fig. 20.2, the average error is 3.19 ns. We can discover that the errors are mostly nanosecond level.

On the basis of pseudorange positioning, use carrier phase smooth P1 and P2, then use the least square method to compute, the average clock error is $6.251205 \times 10^{-7}$ s, which differs 0.95168 ns with the average value of precision clock error.

Figure 20.3 demonstrates that after carrier phase smoothed pseudorange, the difference between calculated clock error and precise clock error, the average difference is 2.61 ns.

The contrast of clock error between pseudorange and carrier phase smoothed pseudorange is reflected in Fig. 20.4. By comparison, the latter's accuracy is higher.

## *20.3.2 Precision Assessment*

In the analysis of pseudorange positioning and carrier phase smoothed pseudorange, we should consider not only the clock error of two methods, but also assess

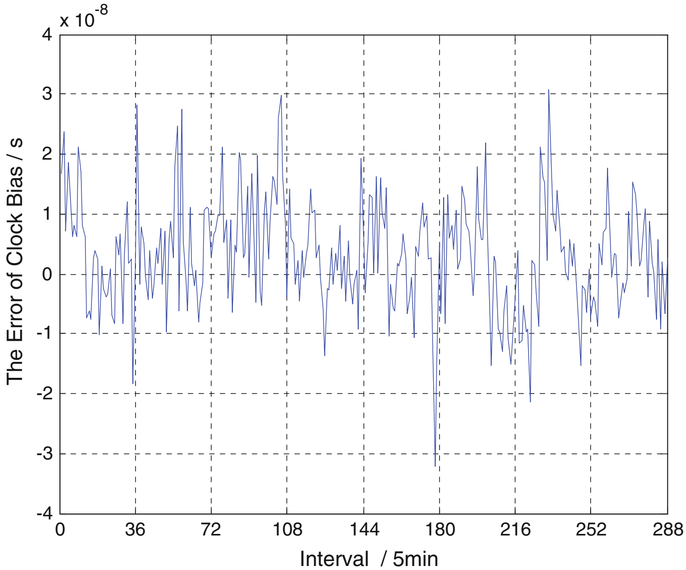


**Fig. 20.2** The error of clock bias by pseudorange positioning

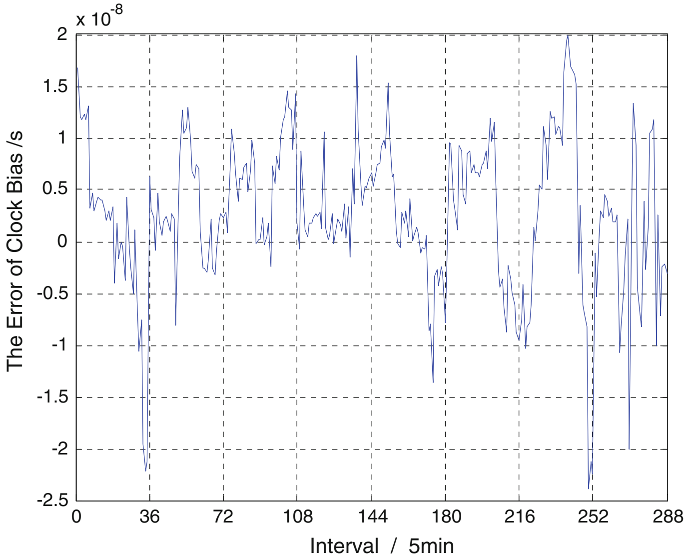


**Fig. 20.3** The error of clock bias by carrier phase smoothed pseudorange

their stability respectively. In this paper, because we only evaluate timing accuracy and clock stability, we can regard internal accord RMS and external accord RMS as assessment standard [2, 7, 8]. Because the receiver of USNO station is atomic

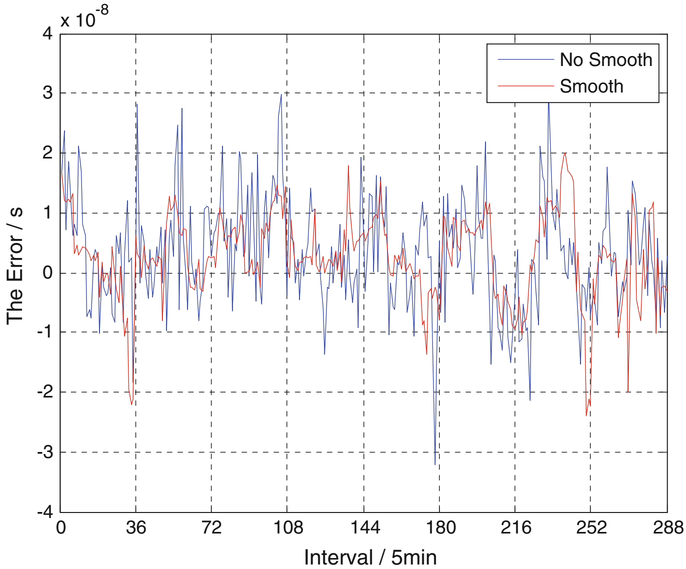


**Fig. 20.4** The difference of error between pseudorange positioning and carrier phase smoothed pseudorange

clock, which clock error is very stable, we can regard the average clock error as reference value. As a result, internal accord RMS can be obtained by difference between calculated clock error and the average value of them. On the other hand, external accord RMS can be obtained by difference between computed clock error and clock error of atomic clock by IGS [8, 5].

$$\text{RMS} = \sqrt{\frac{\Delta^T P \Delta}{n}} \tag{20.10}$$

Where $\Delta$ is difference, n is the total number of samples, P is the weight matrix.

By comparison of two methods, external accord RMS of pseudorange is 9.27 ns, internal accord RMS is 10.22 ns. After carrier phase smoothed pseudorange, external accord RMS is 7.32 ns, internal accord RMS is 7.64 ns.

Detailed precision contrast is illustrated in Table 20.1.

**Table 20.1** The precision index

| Error | Maximum (s) | Minimum (s) | Average (s) | External accord RMS (s) | Internal accord RMS (s) |
|---|---|---|---|---|---|
| Pseudorange positioning | $3.07 \times 10^{-8}$ | $3.21 \times 10^{-8}$ | $3.19 \times 10^{-9}$ | $9.27 \times 10^{-9}$ | $1.02 \times 10^{-8}$ |
| Smoothed pseudorange | $2.01 \times 10^{-8}$ | $2.39 \times 10^{-8}$ | $2.61 \times 10^{-9}$ | $7.32 \times 10^{-9}$ | $7.64 \times 10^{-9}$ |

By comparison, we think that the result of carrier phase smoothed pseudorange is better, the accuracy is higher. The best precision can reach milli nanosecond level.

## 20.4 Conclusions

In this paper, we compared RMS of pseudorange positioning with carrier phase smoothed pseudorange, it illustrates that using carrier phase smoothed paeudorange method can improve the precision of timing effectively, guarantee the stability of clock.

Experimental result indicates that, the precision of timing in single station pseudorange positioning based on GPS system can reach nanosecond level. In addition, the accuracy can be higher when using carrier smoothing pseudorange method.

In this paper, we use broadcast ephemeris data to calculate satellite coordinates and to pseudorange positioning, the precision may be higher if using precise ephemeris data to compute.

**Acknowledgments** This work was carried out through China "Twelfth Five-Year" Plan for National Key Technology R&D Program (contract No.2012BAJ23B01).

## References

1. Hu W, Gao C (2001) GPS principles and applications. China Communications Press, China, pp 60–83
2. Liu R, Yang Z, Zhan X (2012) Research on carrier phase smoothed pseudo-range of compass. J Civil Aviat Univ China 30(3):28–31 (Ch)
3. Holfmann-Wellenhof B, Lichtenegger H, Wasle E (2008) GNSS-global navigation satellite systems—GPS, GLONASS, Galileo & more. Surveying and Mapping Press, Beijing, pp 98–102
4. Xu G (2007) GPS theory, algorithms and applications, 2nd edn. Tsinghua University Press, Beijing, pp P47–P50
5. Plumb JF (2003) Carrier phase time transfer using the global positioning system. University of Colorado, Colorado
6. Larson K (1999) Carrier-phase time transfer. IEEE Trans Ultrason Ferroelectr Freq Control 46(4):1002–1010(Ch)
7. Zhang X, Cheng S, Li X, Guo F (2009) Precise timing using carrier phase smoothed pseudorange from single receiver, J Wuhan Univ 34(4): 463–465 (Ch)
8. Highsmith DE (1988) Precise satellite-to-satellite GPS time transfer in near real-time. Virginia Commonwealth University, Virginia